

Theoretical Estimation of Energy During Magnetic Reconnection in Current Sheet

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ABSTRACT

Various numerical models are given to estimate the energy loss by an eruption process during magnetic reconnection in current sheet and its relation with the energy released during a coronal mass ejection (CME). Here we will discuss the energy range 10^{32} to 10^{33} ergs, by numerical simulations of steady magnetic reconnection in the current sheet. The available magnetic field act as a reservoir of energy and can dissipate its energy to thermal and kinetic energy via the tearing mode instability. An attempt has been made to track the conversion of magnetic energy into kinetic energy, as the magnetic field undergoes reconnection in the current sheet. We have quantified the amount of energy released in an eruptive event in a current sheet and discuss the effects of this energy in generating Coronal Mass Ejection (CME).

Keywords: Magnetic reconnection, Coronal mass ejection, Current sheet and Magnetic energy.

INTRODUCTION

A CME is an outward eruption of large mass and volume of the magnetized plasma in the corona and a flare is an explosive local release of magnetic energy stored previously in a compact region of upper chromospheres or lower corona. Many authors based on their studies supported the idea that the free magnetic energy controls the solar eruption phenomenon (Su *et al.*,

2007; Chen, 2006; Phillip *et al.*, 2005). Some of the sudden magnetic energy release such as flares and CME are probably due to kink instability of filament, torus instability of flux rope or the reconnection between closed field lines (Frobes & Isenberg, 1991; Zhou *et al.*, 2006; Harrison, Devis & Devies, 2009).

Katharine *et al.*, 2010 discussed simulated results of solar eruption in term of several phases.

1. **Helmet steamer relaxation phase** – The magnetic energy increases due to opening of previously closed potential field lines by the solar wind
2. **Shearing phase** - where a shear is introduced to increase magnetic energy about 30×10^{32} ergs to about 50×10^{32} ergs.
3. **Flux cancellation phase** – In this phase the magnetic field continues to increase. Continuous flux cancellation leads to the eruption of flux rope. The magnetic field remain continue to increase also after stopping the flux cancellation but slowly.
4. **Metastable phase** - In this phase the flux rope destabilized and at particular time provides eruption to occur. In this phase also the magnetic energy continues to increase.
5. **Eruption phase** – This occurs when a current sheet develops underneath the ejected flux rope where the simulated magnetic energy decrease from about 60×10^{32} ergs to about 40×10^{32} ergs so loss in magnetic energy converted into kinetic energy, thermal energy and other form of energy.

It is seen that during the first three phases the coronal heating continues but in metastable state as the current sheet form and eruption starts the coronal heating decreases due to eruption because the eruption transport magnetic field farther out into the corona and magnetic field surrounding the flux rope becomes weaker. The largest radiation loss occurs during a flux cancellation phase when prominence like region form. The erupting phase is responsible for loss in magnetic energy. The eruption starts in current sheet when

magnetic reconnection takes place in current sheet and energy released during this process goes into the generation of a flare, CME and Solar Energetic Particle (SEP). Here we discuss the kinetic energy released during the decay of current sheet having a significant role in CME generation.

Flow of Magnetic Energy of a current sheet during reconnection

Currents are generated whenever magnetic field of opposite polarity is brought together. When opposite magnetic fields of same strength approaching each other on the contact surface of the two symmetric flows a current sheet is formed.

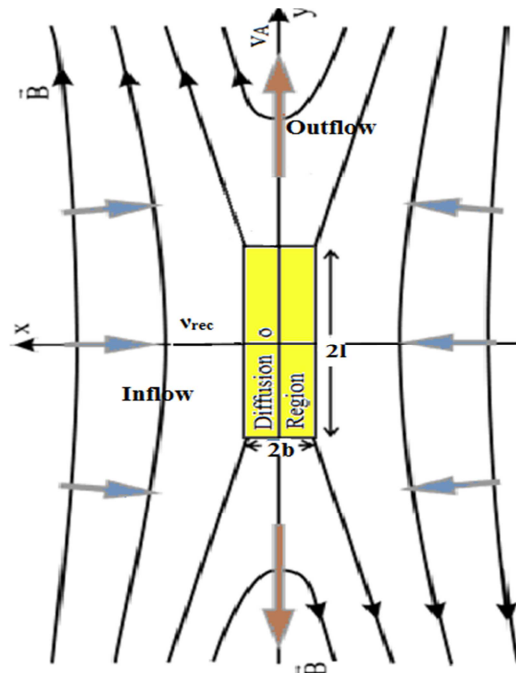


Figure 1: Basic 2D model of a magnetic reconnection process, driven by two oppositely directed inflows (in x-direction), which collide in the diffusion region and create oppositely directed outflows (in y-direction) (Schindler & Hornig 2001).

The Basic MHD Equations

Generally, the Magneto-Hydro Dynamical (MHD) equations can describe the time-dependent evolution of the coronal plasma, but for describing a magnetic field of corona we require stable equilibrium solutions. Such stable equilibrium are expected to be force free ($\mathbf{j} \times \mathbf{B} = 0$), where the plasma density (and the associated pressure p and gravity) have no importance.

From the basic MHD equation in a resistive medium (Aschwanden 2004),

$$\frac{dp}{dt} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P - \rho \mathbf{g} + (\mathbf{j} \times \mathbf{B}) \quad (2)$$

This equation is known as equation of motion where $\mathbf{j} = (c/4\pi) (\nabla \times \mathbf{B})$

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (3)$$

Equation (3) is known as Induction equation where $\eta = c^2/4\pi\sigma$

$$\frac{dP}{dt} + \mathbf{v} \cdot \nabla P + \gamma P (\nabla \cdot \mathbf{v}) = (\gamma - 1) (\nabla \cdot \mathbf{k}_B T) \quad (4)$$

$$P = 2nK_B T$$

Here, ρ , η , $\mathbf{v}$, $\mathbf{B}$, P , n , γ , K_B , c and T are the total mass density, the resistivity coefficient, the velocity, the magnetic field vector, the total thermal pressure, the electron number density, the ratio of specific heats ($\gamma = 5/3$), the Boltzmann constant, the velocity of light and the temperature, respectively.

In static equilibrium, where time dependence vanishes ($d/dt=0$) and flows are constant ($\mathbf{v}=\text{constant}$). Thus the equation of motion becomes

$$-\nabla P - \rho \mathbf{g} + (\mathbf{j} \times \mathbf{B}) = 0 \quad (5)$$

Since horizontal pressure is balanced, we neglect the gravity

$$-\nabla P + (\mathbf{j} \times \mathbf{B}) = 0$$

$$-\nabla P + 1/4\pi[(\nabla \times \mathbf{B}) \times \mathbf{B}] = 0$$

$$-\nabla[P + (B^2/4\pi)] + 1/4\pi[(\mathbf{B} \cdot \nabla)\mathbf{B}] = 0 \quad (6)$$

The first term represents the gradient of total pressure (thermal and magnetic pressure) while the second term represents the magnetic tension. We have considered without bent vertical current sheet where there is no magnetic tension, we can neglect second term

$$-\nabla[P + (B^2/4\pi)] = 0$$

There is no variation in the total pressure with respect to regions.

$$[P + (B^2/4\pi)] = \text{Constant} \quad (7)$$

Total pressure remains constant.

$$[P_i + (B_i^2/4\pi)] = [P_o + (B_o^2/4\pi)] \quad (8)$$

Here P_i , B_i , P_o and B_o are the total pressure and magnetic field inside and outside the current sheet respectively.

Plasma streams along the breath of current sheet (In X-direction)

Let the current sheet of length $2l$, breadth of $2b$ along Y and X axis and it is independently with Z coordinate (Tandberg-Hanssen and Emslie, 1988). the direction of the magnetic field is along Y-axis which is varying linearly in field as a function of the distance from the centre of current sheet.

$$\mathbf{B} = (B_x, B_y, B_z) = B_0(0, x/L, 0)$$

Where L is the magnetic scale height.

$$-\nabla[P+(B^2/4\pi)] + 1/4\pi[(B.\nabla)B] = 0$$

The first term yields a non zero contribution while the second term vanishes ($dB/dt=0$, $dB/dt=0$) so

$$-\nabla[P+(B_o^2/4\pi)] = 0$$

$$-\nabla P = [(B_o/2\pi)(0, x/L, 0)]$$

Which express inward directed magnetic pressure corresponding to the X-direction on the left and anti X-direction on the right. Thus the magnetic pressure drives a lateral inflow of plasma into current sheet, unless the lateral magnetic pressure is balanced by thermal pressure from hotter

plasma inside the current sheet. when change in a magnetic field B and current density J is high inside the current sheet and directed along Y-axis then a reconnection occurs. The maximum current densities increases before magnetic reconnection where initiation continue to increase during reconnection process. This magnetic reconnection can produce

1. CME by the Change in topology
2. Flare by the Conversion magnetic energy into heat and
3. SEP by the acceleration of fast particles (SEP).

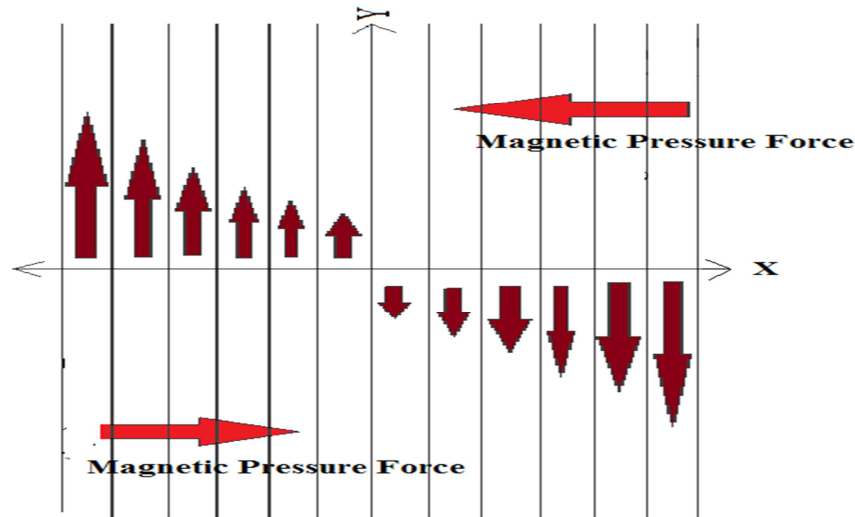


Figure 2: A sketch representing both the opposite magnetic pressure in formation of current sheet.

Plasma streams along the length of current sheet (In Y-direction)

The coronal magnetic field depends on the vertical height and it may be expressed as the inverse of the third power (Aschwanden, 2004).

$$B(h) = B_0 (1 + h/h_D)^{-3},$$

With a mean dipole depth $h_D = 75$ Mm. The photospheric magnetic field strength at their foot points varies in the range of $B_0 = 20 - 200$ G. In corona plasma β ($P_{\text{THERMAL}}/P_{\text{MAGNETIC}}$) < 1 and in general $[P_i(h) + \{B_i^2(h)/4\pi\}] = [P_o(h) + \{B_o^2(h)/4\pi\}]$. Thus it is clear that the region $|Y| > 1$ (outside the current sheet) the total pressure

is smaller, while $|Y| < 1$ (inside the current sheet) achieves higher pressure. If we consider that the total pressure in a current sheet rooted in a high field region in a network ($B_0 < B_i$)

$$[P_i(h) + \{B_i^2(h)/4\pi\}] = P_o(h)$$

The total internal pressure starts to dominate over the external pressure.

$$B_i^2(h)/4\pi \approx P_o(h) - P_i(h) \quad (9)$$

To balance the thermal pressure there is required of decrease of the interior magnetic field and as a result of it fluid is ejected along the field lines (along Y- axis) reducing the built up pressure and allowing the oppositely directed field lines which come closer. A coronal null point ($B = 0$) will occur between the oppositely directed field components where they cancel each other.

Magnetic reconnection inducing Instability in current sheet

A random disturbance of a thin current sheet could occur in the form of current density caused by the lateral inflow of plasma and Lorentz force (opposes the flow).

$$J = c/4\pi(v_A \times B) \approx cB/\mu L \quad (10)$$

$$F_L = j \times B/c$$

In the current sheet where length L is very small and current density j is high enough to produce magnetic reconnection.

For simplicity, we consider the current sheet is vertically symmetric. Thus, the plasma flow into the current sheet from both sides (along X-axis) with inflow speed (v_{rec}) together with frozen in magnetic field lines spreading along the current sheet

upward and downward (along the Y-axis) with outflow speed (v_A) after the field line reconnection. Where the lateral inflows (driven by external forces) will create outflows along the neutral line in an equilibrium situation. The plasma- β parameter becomes larger ($\beta > 1$) in the central region (because $B \rightarrow 0$), so that the plasma can flow across the magnetic field lines, termed as diffusion region, and is channeled into the outflow regions along the neutral boundary. Outside the diffusion region the plasma- β again drops below unity ($\beta < 1$) and the magnetic flux is frozen-in. When driving force (sheared magnetic field) of inflow exceeds the opposing Lorentz force then tearing mode instability (a kind of resistive instability) occurs in the current sheet. Tearing mode instability plays a major role in triggering a solar flare.

If v_A is the Alfvén velocity of fluid along the Y-axis and ρ is the plasma density, where the magnetic pressure is equivalent to the kinetic energy per unit volume. By simply using Bernoulli's theorem;

$$P_o - P_i \approx (1/2) (\rho v_A^2) . \quad (11)$$

In the numerical simulation of Hirose et al. (2001) it is actually found that the major part of the stored magnetic energy is converted into kinetic energy carried away in term of CME, while only a minor part is left for heating of the associated arcade flare.

$$v_A = B / (4\pi\rho)^{1/2} . \quad (12)$$

The Alfvén speed along Y-direction and is height dependence, at a particular location h plasma density $\rho(h)$ and magnetic field $B(h)$ are the function of h . The density and magnetic field vary strongest along the

vertical direction of a magnetic field line. The plasma density $\rho(h)$ decreases near-exponentially with height.

$$\rho(h) \approx \rho_0 \exp(-h/\lambda_T)$$

Where $\rho_0 \approx 2.0 \pm 0.5 \text{ cm}^{-3}$ and thermal scale height $\lambda_T = 47 \text{ Mm} \times (T_e/1 \text{ MK})$.

$$v_A(h) = 2.18 \cdot 10^{11} [B(h)/\{\rho(h)\}^{1/2}] \quad (13)$$

The variations of plasma density and Alfvén velocity with vertical height are shown in Figure 3.

From the continuity equation in the steady state,

$$v_{\text{rec}} l = v_A b$$

Where v_{rec} ($v_{\text{rec}} = \eta/\delta$) is inflow speed and v_A is outflow speed.

$$v_{\text{rec}} = (v_A b)/l$$

$$v_{\text{rec}} = (Bb)/(4\pi\rho)^{1/2}l. \quad (14)$$

According to Maxwell equation

$$4\pi j = \nabla \times B - (\partial E/c\partial t).$$

(Using non-relativistic approximation ($v_A \ll c$) the term $\partial E/c\partial t$ becomes negligible in comparison to $\nabla \times B$ in above equation. The current density j in current sheet becomes

$$j = 1/4\pi (\nabla \times B) \approx cB/4\pi L. \quad (15)$$

When two oppositely directed magnetic fields ($\pm B$) press together then the time of total energy dissipation considered as reconnection time is given as (Parker, 2005)

$$j = (cB/4\pi) (4\pi\eta t_R)^{1/2} \exp(-b^2/4\eta t_R). \quad (16)$$

Using induction equation

$$dB/dt = \nabla \times (v \times B) + \eta \nabla^2 B \quad (17)$$

The equation represent that change in a magnetic field produces transportation

(I term) and diffusion (II term). The ratio of I term to II term gives a Reynold number (R_m)

$$R_m = v_A L / \eta = 4\pi\sigma v_A L / c^2. \quad (18)$$

In most solar phenomenon $R_m > 1$ its value is very high it means that the magnetic field moves with plasma where the magnetic field frozen to plasma energy.

The rate of energy release per unit time (as power) in the current sheet is obtained by,

$$dU/dt = B^2 V / 8\pi t_R \quad (19)$$

Where B is the magnetic field before reconnection, $V = (2l)(2b)\delta = 4lb\delta$ the volume of current sheets, t_R is reconnection time and δ ($v_{\text{rec}} = \eta/\delta$) is the thickness of current sheet.

From equation 15 and equation 16 we get

$$L/(4\pi\eta t_R)^{1/2} = \exp(b^2/4\eta t_R). \quad (20)$$

But $b \gg \eta$ therefore, $\exp(b^2/4\eta t_R) \approx b^2/4\eta t_R$ thus, above equation becomes

$$t_R = \pi/4\eta (b^4/L^2) \quad (21)$$

$$t_R = (16\pi b^4 v_A^2) / 4\eta c^4 \rho^2 R_m^2 \quad (22)$$

From equations 19 and 21, we get

$$dU/dt = \eta^2 c^4 \rho^3 R_m^2 l / 2\pi b^3 v_{\text{rec}} \quad (23)$$

We find out that the energy released from the reconnection in single sheet along the y-direction is not height dependent. We consider $\rho = 10^8 \text{ cm}^{-3}$, $\eta = 7.16 \times 10^{-17}$

($T_e = 2 \text{ MK}$), $v_A = 6.2 \times 10^8 \text{ cm/sec}$, $b = 100 \text{ cm}$, $l = 0.6 \times 10^3 \text{ cm}$, and $R_m = 10^5$

$$dU/dt = 3.96 \times 10^{32} \text{ erg/sec} \approx 4 \times 10^{32} \text{ ergs/sec}$$

Thus during the eruption process the increase in kinetic energy is about 4×10^{32}

ergs/sec which comes due to the energy released by a current sheet during magnetic reconnection.

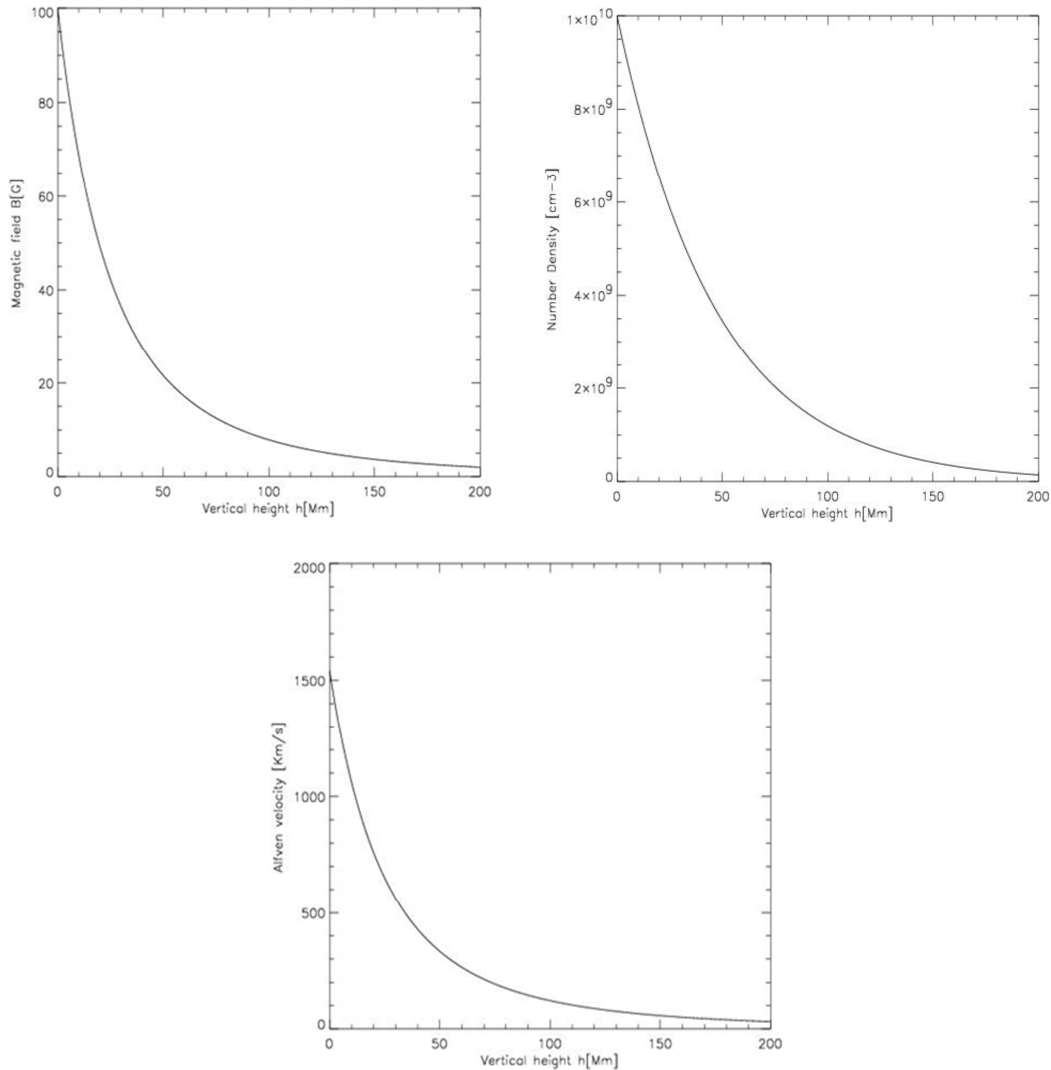


Figure 3. (a) The variation of magnetic field with vertical height. (b) The variation of number density with vertical height. (c) The variation of Alfvén Velocity with vertical height.

RESULT AND DISCUSSION

Numerical simulations of magnetic reconnection have been performed by many

groups (Ugai and Tsuda 1977, Yan *et al.* 1992, Karpen *et al.* 1996, Antiochos *et al.* 2002 and Narain *et al.* 2006). When two

magnetic fields of opposite polarity are brought together a current sheet is produced. The accumulated magnetic free energy of stressed field is released by reconnection. We find that the energy released from the reconnection in single current sheet height independent and have value about 4×10^{32} ergs/sec where the plasma is accelerated by the Lorentz force. As a result, the magnetic energy is mainly transformed into the kinetic energy of the plasma. This energy is responsible to offset the magnetic pressure of flux rope ahead of current sheet. The eruptive flux rope drags magnetic field lines outwards and a CME result from an initial force imbalance due to removal of a part of the plasma in flux rope.

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